Effective capacity of communication systems over κ–μ shadowed fading channels

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The effective capacity of communication systems over generalised κ–μ shadowed fading channels is investigated. A novel and analytical expression for the exact effective capacity is derived in terms of the extended generalised bivariate Meijer's-G function. To intuitively reveal the impact of the system and channel parameters on the effective capacity, closed-form expressions are derived for the effective capacity in the asymptotically high signal-to-noise ratio regime. The results demonstrate that the effective capacity is a monotonically increasing function of channel fading parameters κ and μ as well as the shadowing parameter m, while it decays to zero when the delay constraint $\theta \to \infty$.

Introduction: Shannon's ergodic capacity, which has been extensively studied in the literature, cannot account for the quality of service (QoS) requirements of some emerging real-time applications. Therefore, a novel performance metric is required to ensure delay guarantees for such real-time applications. Motivated by this fact, Wu and Negi [1] proposed the concept of effective capacity (or effective rate, effective throughput) to incorporate the statistical delay QoS guarantees in the capacity of wireless communications, where the effective capacity is defined as the maximum constant arrival rate at the transmitter when guaranteed statistical delay constraints can be satisfied.

Recently, the effective capacity analysis of wireless communication systems has attracted significant research interest, either in independent fading channels [2] or in correlated fading channels [3–5]. However, the presented results only consider small-scale fading environments, such as Rayleigh, Rician, Nakagami-*m*, and κ–μ fading scenarios, while practical wireless channels suffer from both small-scale fading and shadowing simultaneously. Up to now, investigation of the impact of shadowing on the effective capacity is still limited. On the other hand, the κ–μ shadowed fading channel, recently proposed in [6], is a generalised and composite multipath/shadowing fading model. It is based on the assumption that the scattered waves with identical power and the dominant components are subject to the same Nakagami-*m* shadowing fluctuation, and it has been proved to be able to include Rayleigh, one-sided Gaussian, Rician, Rician shadowed, Nakagami-*m*, and κ–μ fading as special cases. Owing to its improved flexibility, the κ–μ distribution can accurately model the statistical variations of wireless communication channels [6].

To this end, we have investigated the effective capacity by deriving the exact expressions for any set of parameters of the κ–μ shadowed fading channel by using the extended generalised bivariate Meijer's-*G* function (EGBMGF) [7, Eqn. (1.1)]. These expressions are generalised and can be reduced to other fading channels.

Effective capacity analysis: As introduced in [1], we suppose that the data arrives in the buffer at a constant rate, and the service process is stationary. A block fading channel is assumed. Analytically, the effective capacity normalised by the bandwidth can be defined as [8, Eqn. (11)]

$$R = -\frac{1}{A}\log_2\left(\mathrm{E}\left\{(1+\gamma)^{-A}\right\}\right) \quad \text{bit/s/Hz} \tag{1}$$

where $\mathrm{E}\{\cdot\}$ refers to the expectation operator, the random variable γ denotes the instantaneous SNR, $A \triangleq \theta TB/\ln 2$ with the asymptotic decay rate of the buffer occupancy θ, the block length T, and the system bandwidth B. From (1), we can find that the effective capacity coincides with the classical Shannon's ergodic capacity when there is no delay constraint as $\theta \to 0$.

For the recently proposed κ–μ shadowed fading channels [6], the probability density function (PDF) of γ is given by [6, Eqn. (4)]

$$f_\gamma(\gamma) = \frac{\mu^\mu m^m (1+\kappa)^\mu}{\Gamma(\mu)\bar{\gamma}(\mu\kappa+m)^m}\left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-1} \times \mathrm{e}^{-\frac{\mu(1+\kappa)\gamma}{\bar{\gamma}}}\,{}_1F_1\left(m, \mu; \frac{\mu^2\kappa(1+\kappa)\gamma}{(\mu\kappa+m)\bar{\gamma}}\right) \tag{2}$$

where m denotes the shaping parameter of the Nakagami-*m RV*, κ represents the power ratio between the dominant components and the scattered waves, μ is the number of clusters, and $\bar{\gamma}$ stands for the average SNR. Moreover, $\Gamma(\cdot)$ represents the Gamma function [9, Eqn. (8.310.1)], and ${}_1F_1(\cdot)$ is the confluent hypergeometric function [9, Eqn. (9.210.1)], respectively. Thus, we can derive the effective capacity in (1) by averaging the SNR γ with the PDF in (2), i.e.

$$R = -\frac{1}{A}\log_2\left(\frac{\mu^\mu m^m(1+\kappa)^\mu}{\Gamma(\mu)\bar{\gamma}^\mu(\mu\kappa+m)^m} I\right) \tag{3}$$

where

$$I = \int_0^\infty (1+\gamma)^{-A}\gamma^{\mu-1}\mathrm{e}^{-\mu(1+\kappa)\gamma/\bar{\gamma}}\,{}_1F_1\left(m, \mu; \frac{\mu^2\kappa(1+\kappa)\gamma}{(\mu\kappa+m)\bar{\gamma}}\right)\mathrm{d}\gamma \tag{4}$$

Note that the integral I in (4) is still not easy to be calculated. To find an analytical solution to the integral I, we express $(1+\gamma)^{-A}$, $\mathrm{e}^{-\mu(1+\kappa)\gamma/\bar{\gamma}}$, and ${}_1F_1\left(m, \mu; \mu^2\kappa(1+\kappa)\gamma/(\mu\kappa+m)\bar{\gamma}\right)$ as Meijer's-*G* functions with the help of [10, Eqns. (10–11)], [9, Eqns. (9.34.8) and (9.212.2)], respectively. Then, by using the integral identity [11, Eqn. (07.34.21.0081.01)], we can obtain

$$I = \frac{\Gamma(\mu)}{\Gamma(A)\Gamma(\mu-m)}\left(\frac{m\mu(1+\kappa)}{(\mu\kappa+m)\bar{\gamma}}\right)^{-\mu} \times G^{0,1;1,1;1,1}_{1,0;1,1;1,2}\left[\begin{matrix}1-\mu\\ -\end{matrix}\middle|\begin{matrix}1-A\\ 0\end{matrix}\middle|\begin{matrix}1+m-\mu\\ 0, 1-\mu\end{matrix}\middle|\frac{(\mu\kappa+m)\bar{\gamma}}{m\mu(1+\kappa)}, \frac{\mu\kappa}{m}\right] \tag{5}$$

The above solution involves an EGBMGF. Although the EGBMGF is not available in standard mathematical packages, an exact and efficient Mathematica implementation has been provided in [12, Table II], which is based on double Mellin-Barnes-type integrals. Substituting (5) into (3) and performing some algebraic simplifications, the exact effective capacity of κ–μ shadowed fading channels can be finally derived as

$$R = -\frac{1}{A}\log_2\left(\frac{1}{\Gamma(A)\Gamma(\mu-m)}\left(\frac{m}{\mu\kappa+m}\right)^{m-\mu} \times G^{0,1;1,1;1,1}_{1,0;1,1;1,2}\left[\begin{matrix}1-\mu\\ -\end{matrix}\middle|\begin{matrix}1-A\\ 0\end{matrix}\middle|\begin{matrix}1+m-\mu\\ 0, 1-\mu\end{matrix}\middle|\frac{(\mu\kappa+m)\bar{\gamma}}{m\mu(1+\kappa)}, \frac{\mu\kappa}{m}\right]\right) \tag{6}$$

Note that the derived effective capacity in (6) is generalised and can be reduced to other fading channels. For example, since κ–μ distribution includes the Rician distribution as a special case by setting $\mu = 1$, (6) reduces to the effective capacity of Rician shadowed fading as

$$R = -\frac{1}{A}\log_2\left(\frac{1}{\Gamma(A)\Gamma(1-m)}\left(\frac{m}{\kappa+m}\right)^{m-1} \times G^{0,1;1,1;1,1}_{1,0;1,1;1,2}\left[\begin{matrix}0\\ -\end{matrix}\middle|\begin{matrix}1-A\\ 0\end{matrix}\middle|\begin{matrix}m\\ 0, 0\end{matrix}\middle|\frac{(\kappa+m)\bar{\gamma}}{m(1+\kappa)}, \frac{\kappa}{m}\right]\right) \tag{7}$$

where κ is now identical to the Rician K factor.

It should be pointed out that although (6) is the exact effective capacity of κ–μ shadowed fading channels, it is only valid for $m \neq \mu$ due to the Gamma function in the denominator. For the special case of $m = \mu$, the PDF of κ–μ shadowed fading in (2) can be simplified to

$$f_\gamma(\gamma) = \frac{m^m}{\Gamma(m)\bar{\gamma}^m}\mathrm{e}^{-m\gamma/\bar{\gamma}}\gamma^{m-1} \tag{8}$$

where the property of [11, Eqn. (07.20.03.0025.01)] has been used. Substituting (8) into (1), using [2, Eqn. (9)] and Kummer's transformation [11, Eqn. (07.33.17.0007.01)], the effective capacity $R_{m=\mu}$ in the case of $m = \mu$ is given by

$$R_{m=\mu} = \log_2\left(\frac{\bar{\gamma}}{m}\right) - \frac{1}{A}\log_2\left(U\left(A; A+1-m; \frac{m}{\bar{\gamma}}\right)\right) \tag{9}$$

where $U(\cdot)$ is the Tricomi hypergeometric function [11, Eqn. (07.33.02.0001.01)].

High-SNR analysis: Although both (6) and (9) are exact expressions for the effective capacity, they cannot provide intuitive insights into the impact of the system and channel parameters on the system performance. Therefore, by considering the initial effective capacity expression (1) and keeping only the dominant term when $\gamma \to \infty$, we can obtain a high-SNR approximation R^{∞} of the effective capacity as

$$R^{\infty}=\log_2\left(\frac{\bar{\gamma}}{\mu(1+\kappa)}\right)-\frac{1}{A}\log_2\left(\frac{\Gamma(\mu-A)}{\Gamma(\mu)}{}_2F_1\left(m,A;\mu;\frac{-\mu\kappa}{m}\right)\right) \quad (10)$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [9, Eqn. (9.100)]. To obtain (10), we have used the integral identity [9, Eqn. (7.522.5)] and a transformation of ${}_2F_1(\cdot)$ [9, Eqn. (9.131.1)]. It is revealed in (10) that the effective capacity grows logarithmically with the average SNR $\bar{\gamma}$ in the high-SNR regime.

Moreover, for the special case of $m=\mu$, the approximated effective capacity $R^{\infty}_{m=\mu}$ in the high-SNR regime can be derived as

$$R^{\infty}_{m=\mu}=\log_2\left(\frac{\bar{\gamma}}{m}\right)-\frac{1}{A}\log_2\left(\frac{\Gamma(m-A)}{\Gamma(m)}\right) \quad (11)$$

where the function ${}_2F_1(\cdot)$ is represented by elementary functions [9, Eqn. (9.121.1)]. It is revealed in (11) that the effective capacity is a monotonically increasing function of μ and m. This is anticipated, since larger values of μ refer to more clusters of multipath components, and larger values of m imply that the shadowing effect decreases.

Numerical results: In Fig. 1, the simulated effective capacity results via Monte-Carlo simulations are compared with the exact and high-SNR approximated analytical expressions provided in (6) and (10) for the effective capacity as a function of the average SNR $\bar{\gamma}$. A precise agreement between the simulated and derived results can be observed, which validates the accuracy of the derived expressions. Moreover, the high-SNR approximations are sufficiently tight and become almost exact when SNR is high, e.g. $\bar{\gamma} > 25$ dB. As anticipated, higher values of m (a weak shadowing condition) tend to result in a higher effective capacity. More importantly, the effective capacity increases when the delay constraint becomes smaller.

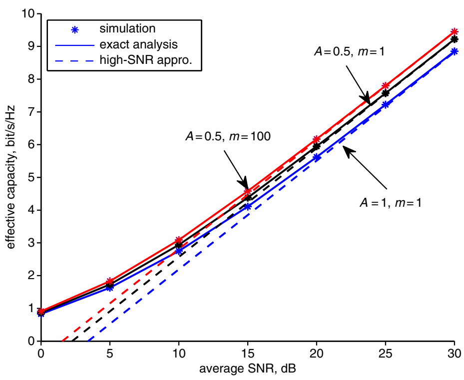

Fig. 1 *Simulated, exact, and high-SNR approximated effective capacity versus average SNR $\bar{\gamma}$ of κ–μ shadowed fading channels with different values of* m *and* A *(κ = 1 and μ = 2)*

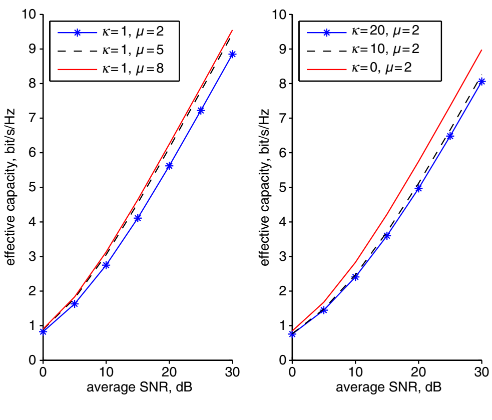

Fig. 2 *Exact effective capacity versus average SNR $\bar{\gamma}$ of κ–μ shadowed fading channels with different values of μ and κ (m = 1 and A = 1)*

Fig. 2 more deeply investigates the impact of small-scale fading parameters κ and μ on the effective capacity of κ–μ shadowed fading channels. As indicated by (11), a higher μ yields a higher effective capacity. However, the gap between the corresponding curves decreases as μ increases, which implies that the effect of μ becomes less pronounced. On similar grounds, the increase of effective capacity is more pronounced for smaller values of κ (less power of dominant components), which reveals that more scattered waves are beneficial for improved effective capacity.

Conclusion: We have presented a detailed analysis of the effective capacity of κ–μ shadowed fading channels. An exact analytical expression has been obtained. Furthermore, we derived a closed-form expression in the high-SNR regime. The derived results reveal that a performance gain can be obtained by loosening the delay constraint θ as well as in a propagation environment with larger values of μ and m. Moreover, higher power of dominant components (larger values of κ) tends to decrease the effective capacity.

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One or more of the Figures in this Letter are available in colour online.

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