Hysteresis modeling in the MATLAB/Power System Blockset

Silvano Casoria,∗, Gilbert Sybille,a,1, Patrice Brunelle,b,2

a Institut de recherche d’Hydro-Québec (IREQ), 1800 boul. Lionel-Boulet, Varennes, Que., Canada J3X 1S1
b TransÉnergie Technologies Inc., 740 Notre-Dame Ouest, Montréal, Que., Canada H3C 3X6

Abstract
This paper presents a static hysteresis model for the saturable transformer blocks in the Power System Blockset (PSB), a MATLAB/Simulink®-based simulation tool. This model will be available in version 3.0 of the PSB (also called SimPowerSystems) and offered as an alternative to the actual core saturation characteristic. The effect of hysteresis and eddy losses are of significance in the studies of: sub harmonic; residual flux effects on the transformer inrush currents; ferroresonance phenomena; harmonics generated by half-cycle saturation.

The static model of hysteresis defines the relation between the flux $\Phi$ and the magnetization current (i.e. the current through the nonlinear inductance in the model) that is equal to the total excitation current measured in dc (when the eddy current losses are not present). It is useful under transient conditions and can represent minor loops.

The principal points are: in transient conditions an oscillating current will produce minor asymmetrical loops; inside a minor loop the excitation curve depends only on the last two reversal points, and each curve tend to return to the reversal point previous to the last.

A hysteresis design tool consisting of graphical user interface (GUI) allows precise adjustment to any major loop or saturation characteristic. An animation option of the GUI permits an open loop simulation so to visualize the operating point displacement within the major loop and the formation of the minor loops.

Finally, an example case is described to illustrate remanent flux and inrush current at transformer energization.

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1. Introduction

THE MATLAB/Power System Blockset (PSB) is a electromagnetic transient program capable of representing typical power equipment such as transformers, lines, machines, and power electronics to simulate...
power systems [1,2]. The blockset uses the Simulink® graphical environment, allowing a model to be built using simple click and drag procedures.

The PSB saturable transformer model, has the magnetizing characteristics of the core modeled by a resistance ($R_m$) simulating the core active losses (eddy currents and hysteresis losses) and a saturable inductance in parallel. The saturation characteristic is specified as a piecewise linear characteristic of a nonlinear relation between the flux and the magnetization current. For increased accuracy, modeling of the hysteresis phenomenon is introduced in version 2.3.

The effect of hysteresis and eddy losses are of significance in the studies of: subsynchronous oscillations where system losses appears to be determinant; residual flux effects on the transformer inrush currents; ferroresonance phenomena; harmonics generated by half-cycle saturation. Furthermore, the hysteresis representation can be of importance in the implementation of instrument transformers models such as current transformers (CT) and capacitor coupled voltage transformers (CCVT) [3].

Saturation and hysteresis modeling have been the subject of many papers with different approaches depending on the application. Emphasis, may be put on the empirical or on the analytical nature of the expressions (e.g. exponential, hyperbolae, polynomials, arctangent) used for the operating point ($\Phi$, $I$) trajectory representation.

Importance can also be given to the algorithm (e.g. iterative for more precision, non iterative for speed of execution) that will dictate the behavior of the operating point trajectory during transients (e.g. for the formation of the minor loops).

The proposed model uses a semi-empirical characteristic, by having an arctangent analytical expression representing the operating point trajectories [4]. The expression parameters are deduced by curve fitting empirical data defining the major loop or the single-valued saturation characteristic. A hysteresis design tool consisting of a graphical user interface (GUI) allows precise adjustment to any hysteresis major loop or saturation characteristic. The algorithm selected for modeling the trajectories behavior and the formation of the hysteresis minor loops is based on the one used in the electromagnetic transients program (EMTP) [5]. We have improved the algorithm by adding the possibility of unlimited number of embedded minor loops formation.

2. Hysteresis model

The static model of hysteresis defines the relation between the flux ($\Phi$) and the magnetization current (i.e. the current through the nonlinear inductance in the model) that is equal to the total excitation current measured in dc (when the eddy current losses are not present).

The fundamental characteristics of the model [6] are the following.

1. The symmetric variation of the flux produces a symmetrical current variation between $-I_{\text{max}}$ and $I_{\text{max}}$, resulting in a symmetrical hysteresis loop whose shape and surface depend on the value of $\Phi_{\text{max}}$ as shown in Fig. 1. The major loop is produced when $\Phi_{\text{max}}$ is equal to the saturation flux ($\Phi_s$). Beyond that point the characteristic reduces to a single-valued saturation characteristic, which is asymptotic to the air core inductance ($L_s$). The remanent flux ($\Phi_r$), the coercive current ($I_c$) and its slope ($d\Phi/dI$) depend on the core material.

2. In transient conditions an oscillating magnetizing current will produce minor asymmetrical loops, as shown in Fig. 2 and all points of operation are assumed to be within the major loop. Loops once closed have no more influence on the subsequent evolution.
Fig. 1. Hysteresis loops.

Fig. 2. Internal minor loops.
3. Inside a minor loop, the magnetizing curve depends only on the last two reversal points, and each curve tends to return to the reversal point previous-to-the last (e.g. the evolution 1–2–3–4–3–5–2–6–1) in Fig. 2.

2.1. Arctangent basic characteristic

The minor loops are derived from the major loop ascending (i.e. \( \frac{d\Phi}{dt} > 0 \)) or descending (i.e. \( \frac{d\Phi}{dt} < 0 \)) trajectories applying simple rules adapted from [4]. The basic analytical expression \( \Phi(I) \) representing a major loop half-cycle is given by (1)

\[
\Phi = -\text{sgn}[a \times \arctan(- \text{sgn} \times b \times I + c) - \text{sgn} \times \alpha \times I + e]
\]

where \( \text{sgn} = 1 \) for an ascending trajectory and \( -1 \) for a descending trajectory.

The parameters \( \alpha, c, b \) are related to the residual (remanent) flux \( F_r \), coercive current \( I_c \), and slope \( \frac{dF}{dI} \) at the coercive current as shown in Fig. 1. The parameters \( a \) and \( e \) are related to the saturation point coordinates \( I_s \) and \( \Phi_s \). These parameters are calculated by solving a set of non-linear equations during the parameterization step, presented in the next section, using the hysteresis design tool.

During simulation, the model uses the inverse relation \( I(\Phi) \) since the independent or input variable is the flux. The inverse relation is defined as a series of \( N \) equidistant points connected by line segments. A table containing the coordinates of these points \( (I_n, \Phi_n) \) and the segments slope is defined by solving iteratively a set of non-linear equations.

2.2. Minor loops formation

To generate the closed minor loops we apply the fundamental characteristic three given before. It stipulates that each curve tends to return to the previous-to-last point of reversal, e.g. reversal point 1 on the ascending half of the major loop in Fig. 3. Effectively, after detecting a reversal at point 2, a new trajectory 2–1 is calculated. First, \( D_{\text{max}} \), which is the distance between the reversal point 2 and the appropriate half of the major loop is calculated. In addition, the vertical distance \( D_{\text{min}} \) between 1 and the major loop is defined (0 in this example). By assuming, that the vertical distance \( D \) decreases linearly with \( \Phi \), that is the independent variable, all the points of the trajectory are then defined. A linear relation \( D(\Phi) \) of \( D \) as function of the flux is found at each reversal point and is stored in memory along with the coordinates of this point. Memorization is important in order to represent properly the behavior of embedded loops as the evolution on Fig. 2 illustrates.

When a reversal point is overtaken, like point 3 in the evolution 3–4–3–5, the parameters of the trajectories 3–4–3 are discarded and the curve continues on the trajectory 2–1, already in memory. Point 1 will not be reached since the flux reverses at point 5. Next, the reversal point 2 is overtaken in the evolution 5–2–6, and the parameters of the trajectories constituting the loop 2–5–2 are discarded. Finally, the curve continues on the trajectory 6–1.

2.3. Special conditions

The assumption of linear dependence of \( D(\Phi) \) on \( \Phi \) can lead to the difficulty where for certain values of \( F \), the minor loop trajectory would lie outside the major loop, as in Fig. 4. To avoid this, the
Fig. 3. Formation of minor loop trajectory.

Fig. 4. Solution to a probable illegal operating condition.
trajectory is tested and constrained to follow the major loop. This situation arises only near the saturation points and the small correction introduced to the trajectory would not significantly affect the model accuracy.

Two other operating conditions should be identified in order to avoid generating superfluous internal loops or trajectories. In the hysteresis design tool, two tolerances can be adjusted: $\text{TOL}_F$ (percent of $F_s$) and $\text{TOL}_I$ (percent of $I_c$). These parameters trigger detection of one of the following two conditions.

1. Closed minor loops in steady-state.
2. Very small minor loops, whose internal surface may be neglected.

The first condition will be assumed if the $\Phi$ value of the actual reversal point is in close proximity to the previous-to-last reversal point $\Phi$ value, i.e. within an adjusted tolerance $\text{TOL}_F$ (default = 0.1% of $\Phi_s$). In that case, no new trajectory is calculated and the operating point will remain on the same loop. This may be illustrated by the potential evolution 3–4–3–4 in Fig. 2, forming a closed loop in steady-state.

The second condition will be assumed when the distance between the $I$ coordinate of the actual point of reversal and the previous-to-last is less than a threshold value $\text{TOL}_I$ (default = 0% of $I_c$ or null). If such a condition is detected, the evolution within these two points will follow a trajectory defined by a line segment. This is illustrated in Fig. 5 by the two series, A and B, of imbedded minor loops, with similar current evolution, only shifted vertically. In series B (evolution 1–2–3–4–3–1), the reference is adjusted to a high value and consequently the condition is not detected. In series A, the reference is such that the condition is detected and the procedure applied.

![Fig. 5. Very small minor loops (series B) are approximated by line segments (series A).](image-url)
2.4. Initial trajectory

At the beginning of the simulation the initial trajectory must be specified. The trajectory depends on the operating point past evolution, i.e. by the last two reversal points. Assuming that the last reversal point is situated on the major loop, the initial trajectory would be determined by the residual flux value and on the reversal point position on either the ascending or descending half. An increasing flux initially indicates that the reversal point is on the descending half and vice versa for a decreasing flux. Fig. 6, illustrates the two possibilities (trajectory A and B) for a residual flux of 0.1 pu.

The direction of the flux can be determined during the steady-state solution when the transformer is connected to the network, otherwise an increasing flux is assumed. The user may specify the residual flux value or it is automatically adjusted so that the simulation starts in steady-state.

3. Model parameterization

The parameters of the hysteresis major loop and the single-valued saturation characteristic are specified in the hysteresis design tool user interface available with the Power System Blockset. The remanent flux, the saturation flux and current points, the coercive current, and the slope of the flux at the coercive current point can be entered in pu or in SI units. In addition, two vectors of current-flux pair of points specify the saturation characteristic. The graphical user interface tool is illustrated in Fig. 7.

The hysteresis characteristic is displayed in the interface and it can be updated at any time during the specification of the parameters. The flux and currents parameters are also displayed on the graph for a
fast and convenient design. The characteristics are saved in MATLAB data files for a later use with the hysteresis transformer model. As many characteristics as necessary can be generated and saved in order to be used by different transformers in the simulation model.

A special interface option allows the user to view an interactive simulation of the minor loop formation. The hysteresis design tool performs an animation of the flux–current evolution based on the trajectory initial and final flux as specified by the user.

The two tolerances \( TOL_F \) and \( TOL_I \) can also be specified in the interface and they are saved in the data file with the other hysteresis parameters.

4. Model implementation

The hysteresis and saturation characteristic of the saturable transformer of PSB is built with Simulink® blocks. Fig. 8 illustrates the corresponding electrical model for one phase implemented into the Power
The winding resistances and leakage inductances as well as the core active losses resistance $R_m$ of the saturable transformer are modeled by state variables. This state-space model is incorporated into the global linear state-space system of the rest of the network connected to the transformer terminals.

The magnetizing branch is modeled as a current source where the current is computed from the flux obtained by integrating the voltage across the magnetization branch ($V_{mag}$). The block diagram of Fig. 9 shows the Simulink® implementation of the model.

The Power System Blockset provides the magnetization voltage of the magnetizing branch to the hysteresis model at each simulation time step and the model returns the corresponding current that is injected in the rest of the network system.

The major hysteresis loop and the saturation characteristic are implemented with look-up tables for a fast and efficient simulation. The upper and lower major trajectories are modeled separately and are controlled by a Simulink® switch based on the sign of the flux. A second switch allows the model to pass from the major loop calculation to the inner or minor hysteresis loops calculation.

The Simulink® S-function is in the heart of this process. Its algorithm controls the transition between the major and the minor hysteresis loops, calculates the minor loops trajectories, and outputs the magnetization current.

Fig. 8. Electrical model of the Power System Blockset saturable transformer.

Fig. 9. Simulink® model of the hysteresis implemented in the Power System Blockset.
The parameters that define the hysteresis model and also the saturation region are passed to the S-function via a MATLAB data file. When the simulation is started in Simulink®, the S-function first calculates the initial trajectory based on the initial flux and on the sign of the initial magnetization voltage across the magnetizing branch.

The S-function continuously monitors the flux and controls the Simulink® switches when the flux leaves the minor loop region to enter in the saturation region or when the flux leaves the major hysteresis loop to enter in the minor loop zone. The function also calculates the trajectories and evolution of the internal minor loops and outputs the current as a function of the input flux.

5. Applications example

Fig. 10 illustrates the simulation of the hysteresis in a saturable transformer block of the PSB. Phase A of the primary winding is connected on a 500 kV network, and the secondary is not connected to any load. The transformer is rated 500/230 kV, 450 MVA (150 MVA per phase) and the flux–current saturation characteristic of the transformer was modeled with the hysteresis design tool. The air core inductance, i.e. the last segment of the saturation characteristic, is 0.4 pu.

A programmable source is used to vary the internal voltage of the equivalent 500 kV network. During the first three cycles the source voltage is programmed at 0.8 pu. Then at 50 ms, the voltage is increased to 1.1 pu. In order to illustrate remanent flux and inrush current at transformer energization, the circuit breaker initially closed is first opened at 0.1 s, and then it is reclosed at 0.15 s.

The initial flux in the transformer is set at zero and the source phase angle is adjusted at 90° so that the flux remains symmetrical around zero when simulation is started.

The following observations can be made on the simulation results of Fig. 11.

1. From 0 to 0.05 s, the voltage and flux peak values are at 0.8 pu. Typical square wave of magnetization current can be observed. As no remanent flux was specified, the magnetization current and the flux are symmetrical. The flux travels on minor loops.

2. From 0.05 to 0.1 s, the voltage is raised to 1.1 pu. The flux now reaches 1.1 pu and it travels on the main hysteresis loop. Current pulses appear on the magnetization current indicating beginning of saturation.

3. At first zero crossing after the breaker opening order, the current is interrupted and a flux of 0.83 pu stays trapped in the transformer core.

4. The breaker is reclosed at \( t = 9 \) cycles, at a zero crossing of source voltage, producing an additional flux offset of approximately 1.02 pu. The peak flux now reaches 1.85 pu, driving the transformer into the saturated region. Peak magnetization current is 0.81 pu.

Fig. 10. Power System Blockset diagram of the application example.
6. Conclusions

The paper presents a model of the static hysteresis effect of the ferro-magnetic core in power transformers. This model is suitable for simulation of switching transients and has the capability of representing minor loops. It is conceived according to principal characteristics of transformer core obtained experimentally and given in the literature. The model, that can also represent the saturation characteristic, has been implemented in the latest MATLAB/Power System Blockset, and included as an option in the saturable transformer model.

The model is based on the characteristics of the major hysteresis loop out of which the internal trajectories are defined using the translation principal and a linear compensation to generate closed loops. An arctangent relation between the flux and the exciting current is defined. The expression parameters are deduced by curve fitting empirical data defining the major loop or the single-valued saturation characteristic. A hysteresis design tool consisting of a graphical user interface allows precise adjustment to any hysteresis major loop or saturation characteristic.

The major hysteresis loop and the saturation characteristic are implemented with look-up tables for a fast an efficient simulation. A Simulink® S-function contains the algorithm controlling the transition between the major loop and the internal minor loops, calculates the minor loops trajectories, and outputs the magnetization current.

An application example illustrates the simulation of hysteresis in a transformer with no load by showing the remanent flux at isolation and the inrush current at energization.
References


